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New Entropy Measures Based on Neutrosophic Set and Their Applications to Multi-Criteria Decision Making

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Abstract: Our aim in this work is to obtain two new entropy measures for single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs). Moreover, we give the essential properties of the proposed entropies. Finally, we introduce a numerical example to show that the entropy measures are more reliable and reasonable for representing the degree of uncertainty.

Neutrosophic Küme Üzerinde Yeni Entropi Ölçüsü ve Çok Kriterli Karar Verme Uygulamaları

Anahtar Kelimeler
Neutrosophic küme,
Tek-değerli neutrosophic küme,
Aralık-değerli neutrosophic küme,
Entropi ölçüsü,
Karar verme

Özet: Bu çalışmadaki amacımız, tek-değerli neutrosophic kümeler (SVNSs) ve aralık-değerli neutrosophic kümeler (INSs) için iki yeni entropi ölçüsü oluşturmaktır. Buna ek olarak, oluşturulan entropilerin temel özelliklerini gösterdik. Son olarak, oluşturulan entropi ölçülerinin belirsizlik derecesini temsil edebilmede daha makul ve güvenilir olduklarını gösteren bir sayısal örnek verdik.

1. Introduction

Neutrosophy is a branch of philosophy which associates the logical knowledge, set theory, philosophy and probability. Smarandache [1,2] introduced the neutrosophic sets (NSs). Unlike the fuzzy sets (FSs) and intuitionistic fuzzy sets (IFSs), an NS is formed that the truth-membership function (TMF), the indeterminacy-membership function (IMF) and the falsity-membership function (FMF). Although the combined uncertainty is dependent on the belongingness and non-belongingness degrees of existing sets, the uncertainty presented here is independent on the truth and falsity values. The structure of NSs is not convenient to implement real-life situations. Thus, Wang et al. [3,4] improved SVNSs and INSs, which are generalization of NSs.

Entropy measure is a very important concept for measuring fuzziness degree or uncertain information in fuzzy set theory. Therefore, it has attracted considerable attention during the recent years. In 1965, Zadeh [5] presented the entropy measure for FSs. De Luca and Termini [6] first gave axiomatic structure to determine the fuzziness degree of fuzzy set and introduced the entropy of FS based on Shannon's function in [7]. Bustince and Burrillo [8] introduced the distance measure between IFSs and entropy for IFS. Szmidt and Kacprzyk [9] proposed entropy for IFSs, which based on an extension of fuzzy entropy axioms of De Luca and Termini's [6] work. Ye [10] introduced entropy measure for inter valued intuitionistic fuzzy sets (IVIFS). Wei et al. [11] defined entropy measure for IVIFS. Majumdar and Samanta [12] gived the entropy measure for SVNSs and proposed its some properties. Aydoğdu [13,14] introduced similarity and entropy measure for SVNSs and INSs. Ye and Du [15] proposed distances, similarity and entropy measures for INSs. Ye [16-17] established multi-criteria decision-making (MCDM) method under SVNSs. Ye [18] introduced cross entropy for SVNSs and INSs and gave MCDM methods. Tian et al. [19] proposed MCDM method under INSs. Şahin [20] established a cross entropy measure of INSs and introduced MCDM methods under INSs. Peng and Dai [21-22] gave an analysis of neutrosophic-related research published from 1998 to 2017, and introduced distance measure and similarity measure

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for SVNSs and proposed MCDM methods. In this study, we define two new entropy measures for SVNSs and INSs, respectively. Then we apply the entropy measure of SVNSs to solve an MCDM problem, which the attribute values are elements of SVNSs. We introduce an example to show the convenience of the introduced method in its practical applications.

2. Material and Method

This section gives a brief outline of NSs, SVNSs and INSs.

Definition 2.1. [2] Let $\mathfrak{U}$ be a universal set, then a NS is defined as:

$$S = \{\langle y, t_S(y), i_S(y), f_S(y)\rangle : y \in \mathfrak{U}\},$$

which is typified by a TMF, an IMF and a FMF, respectively. Here the TMF, the IMF and the FMF are functions from $\mathfrak{U}$ to non-standard unit interval $]^-0, 1^+[$.

There is not any limitation on the sum of membership functions, so

$$^-0 \leq \sup t_S(y) + \sup i_S(y) + \sup f_S(y) \leq 3^+.$$

We now give definition of SVNS.

Definition 2.2. [3] Let $\mathfrak{U}$ be a universal set, then a SVNS S in $\mathfrak{U}$ is defined as:

$$S = \{\langle y, t_S(y), i_S(y), f_S(y)\rangle : y \in \mathfrak{U}\},$$

where $t_S: \mathfrak{U} \to [0,1]$, $i_S: \mathfrak{U} \to [0,1]$ and $f_S: \mathfrak{U} \to [0,1]$.

The values $t_S(y), i_S(y)$ and $f_S(y)$ denote the truth-membership degree (TMD), the indeterminacy-membership degree (IMD) and the falsity-membership degree (FMD) of y, respectively, and the sum of the TMD, IMD and FMD is in the interval [0,3].

The set $\mathcal{G}$ is denoted set of all the SVNSs in $\mathfrak{U}$. We denote the single valued neutrosophic number (SVN) by $S = \langle t_S, i_S, f_S\rangle$ for convenience.

Definition 2.3. Let S and T be two SVNSs. The intersection of S and T, denoted by $N = S \cap T$, is defined by

$$t_N(y) = \min\{t_S(y), t_T(y)\}$$
$$i_N(y) = \min\{i_S(y), i_T(y)\}$$
$$f_N(y) = \max\{f_S(y), f_T(y)\}$$

for all $y \in \mathfrak{U}$.

Definition 2.4. Let S and T be two SVNSs. The union of S and T is a SVNS U, denoted by $U = S \cup T$, is defined as

$$t_U(y) = \max\{t_S(y), t_T(y)\}$$
$$i_U(y) = \max\{i_S(y), i_T(y)\}$$
$$f_U(y) = \min\{f_S(y), f_T(y)\}$$

for all $y \in \mathfrak{U}$.

Definition 2.5. The complement of SVNS S is denoted by S^c and is defined by

$$t_{S^c}(y) = f_S(y)$$
$$i_{S^c}(y) = 1 - i_S(y)$$
$$f_{S^c}(y) = t_S(y)$$

for all $y \in \mathfrak{U}$.

Definition 2.6. Let S and T be two SVNSs. Then S is contained the T, is denoted $S \subseteq T$, if and only if

$$t_S(y) \leq t_T(y)$$
$$i_S(y) \leq i_T(y)$$
$$f_S(y) \geq f_T(y)$$

for all $y \in \mathfrak{U}$.

Wang et al. [4] introduced INSs, is characterized by a truth membership interval (TMI), an indeterminacy membership interval (IMI) and a false membership interval (FMI) neutrosophic set. It is used to deal with uncertainty in fields of scientific, engineering environment, etc.

Definition 2.7. [4] Let $\mathfrak{U}$ be universal set. The set of all closed subsets of [0,1] is denoted by $\boldsymbol{I}$. An INS $N \in \mathfrak{U}$ is characterized by a TMF $\boldsymbol{t}_N: \mathfrak{U} \to \boldsymbol{I}$, a IMF $\boldsymbol{i}_N: \mathfrak{U} \to \boldsymbol{I}$ and a FMF $\boldsymbol{f}_N: \mathfrak{U} \to \boldsymbol{I}$, with the form

$$N = \{\langle y, \boldsymbol{t}_N(y), \boldsymbol{i}_N(y), \boldsymbol{f}_N(y)\rangle : y \in \mathfrak{U}\}.$$

Let $\boldsymbol{t}_N(y) = [t_N^l(y), t_N^u(y)]$, $\boldsymbol{i}_N(y) = [i_N^l(y), i_N^u(y)]$ and $\boldsymbol{f}_N(y) = [f_N^l(y), f_N^u(y)]$, then INS N is

$$\{\langle y, [t_N^l(y), t_N^u(y)], [i_N^l(y), i_N^u(y)], [f_N^l(y), f_N^u(y)]\rangle : y \in \mathfrak{U}\}$$

with, $0 \leq \sup t_N^u(y) + \sup i_N^u(y) + \sup f_N^u(y) \leq 3$ for all $y \in \mathfrak{U}$. It is clear that an INS is NS.

Definition 2.8. [4] Let N and M be two INSs. The intersection of N and M is INS K, denoted by $K = N \cap M$, is defined as

$$t_K^l(y) = \min\{t_N^l(y), t_M^l(y)\}$$
$$t_K^u(y) = \min\{t_N^u(y), t_M^u(y)\}$$
$$i_K^l(y) = \max\{i_N^l(y), i_M^l(y)\}$$
$$i_K^u(y) = \max\{i_N^u(y), i_M^u(y)\}$$
$$f_K^l(y) = \max\{f_N^l(y), f_M^l(y)\}$$
$$f_K^u(y) = \max\{f_N^u(y), f_M^u(y)\}$$

for all $y \in \mathfrak{U}$.

Definition 2.9. [4] Let N and M be two INSs. The union of N and M is an INS U, is written by $U = N \cup M$, is defined as follow

$$t_U^l(y) = \max\{t_N^l(y), t_M^l(y)\}$$
$$t_U^u(y) = \max\{t_N^u(y), t_M^u(y)\}$$
$$i_U^l(y) = \min\{i_N^l(y), i_M^l(y)\}$$
$$i_U^u(y) = \min\{i_N^u(y), i_M^u(y)\}$$
$$f_U^l(y) = \min\{f_N^l(y), f_M^l(y)\}$$
$$f_U^u(y) = \min\{f_N^u(y), f_M^u(y)\}$$

for all $y \in \mathfrak{U}$.

Definition 2.10. [4] Let Nbe INS. Denote by N^c the complement of N and the INS N^c is defined by

$$t_{N^c}(y) = f_N(y)$$
$$f_{N^c}(y) = t_N(y)$$
$$i_{N^c}^l(y) = 1 - i_N^u(y)$$
$$i_{N^c}^u(y) = 1 - i_N^l(y)$$

for all $y \in \mathfrak{U}$.

Definition 2.11. [4] An INS M contain in the other INS N, is denoted by $N \subseteq M$, if and only if

$$t_N^l(y) \leq t_M^l(y);\ t_N^u(y) \leq t_M^u(y)$$
$$i_N^l(y) \geq i_M^l(y);\ i_N^u(y) \geq i_M^u(y)$$
$$f_N^l(y) \geq f_M^l(y);\ f_N^u(y) \geq f_M^u(y)$$

for all $y \in \mathfrak{U}$.

3. Results

Definition 3.1. [12] Let $\mathcal{G}$ be all SVNSs on $\mathfrak{U}$ and $S \in \mathcal{G}$. An entropy on SVNSs is a function $E_{\mathcal{G}}: \mathcal{G} \to [0,1]$ which satisfying:

i. $E_{\mathcal{G}}(S) = 0$ if S is crisp set
ii. $E_{\mathcal{G}}(S) = 1$ if $\big(t_S(y), i_S(y), f_S(y)\big) = (0.5,0.5,0.5)$ for all $y \in \mathfrak{U}$
iii. $E_{\mathcal{G}}(S) \geq E_{\mathcal{G}}(T)$ if $S \subset T$, i.e., $t_S(y) \leq t_T(y)$, $f_S(y) \geq f_T(y)$, $i_S(y) \leq i_T(y)$ for all $y \in \mathfrak{U}$
iv. $E_{\mathcal{G}}(S) = E_{\mathcal{G}}(S^c)$ for all $S \in \mathcal{G}$.

Definition 3.2. Let S be a SVNS. Then the entropy of S is,

$$E_{\mathcal{G}}(S) = \frac{1}{n}\sum_{i=1}^{n} \frac{2 - |t_S(y_i) - f_S(y_i)| - |i_S(y_i) - i_{S^c}(y_i)|}{2 + |t_S(y_i) - f_S(y_i)| + |i_S(y_i) - i_{S^c}(y_i)|}$$

for all $y_i \in \mathfrak{U}$.

Theorem 3.3. The SVN entropy of $E_{\mathcal{G}}(S)$ is an entropy measure for SVNSs.

Proof: We show that the $E_{\mathcal{G}}(S)$ satisfies the conditions $i - vi$ in Definition 3.1.

i. When S is a crisp set, i.e., $t_S(y_i) = 0, i_S(y_i) = 0$, $f_S(y_i) = 1$ or $t_S(y_i) = 1$, $i_S(y_i) = 0$, $f_S(y_i) = 0$, for all $y_i \in \mathfrak{U}$. It is clear that $E_{\mathcal{G}}(S) = 0$.
ii. Let $\big(t_S(y), i_S(y), f_S(y)\big) = (0.5,0.5,0.5)$. Then

$$E_{\mathcal{G}}(S) = \frac{1}{n}\sum_{i=1}^{n} \frac{2 - |0.5 - 0.5| - |0.5 - 0.5|}{2 + |0.5 - 0.5| + |0.5 - 0.5|} = \frac{1}{n}\sum_{i=1}^{n} 1 = 1.$$

iii. If $S \subset T$, then $t_S(y) \leq t_T(y)$, $f_S(y) \geq f_T(y)$ and $i_S(y) \leq i_T(y)$ for all $y \in \mathfrak{U}$. So $t_S(y_i) - f_S(y_i) \leq t_T(y_i) - f_T(y_i)$ and $i_S(y) - i_{S^c}(y_i) \leq i_T(y) - i_{T^c}(y_i)$. Since $|t_S(y_i) - f_S(y_i)| + |i_S(y_i) - i_{S^c}(y_i)| \leq |t_T(y_i) - f_T(y_i)| + |i_T(y_i) - i_{T^c}(y_i)|$, $E_{\mathcal{G}}(S) \geq E_{\mathcal{G}}(T)$.

iv. Since $t_{S^c}(y) = f_S(y)$, $i_{S^c}(y) = 1 - i_S(y)$ and $f_{S^c}(y) = t_S(y)$, it is clear that $E_{\mathcal{G}}(S) = E_{\mathcal{G}}(S^c)$.

The proof is completed.

In many practical situations, one should be considered the weight of each element $y \in \mathfrak{U}$. For instance, the considered attribute has generally different importance in MADM problems. Herewith its is appointed with different weights. Assume that the weights $\omega = (\omega_1, \omega_2, \dots \omega_n)^T$ with $\omega_j \in [0,1]$, $\sum_{i=1}^{n} \omega_i = 1$. Then weighted entropy measure is defined as follows:

$$E_{\omega\mathcal{G}}(S) = \frac{1}{n}\sum_{i=1}^{n} \omega_i \left(\frac{2 - |t_S(y_i) - f_S(y_i)| - |i_S(y_i) - i_{S^c}(y_i)|}{2 + |t_S(y_i) - f_S(y_i)| + |i_S(y_i) - i_{S^c}(y_i)|}\right).$$

Definition 3.4. Let $\mathcal{J}$ be all INSs on $\mathfrak{U}$ and $N \in \mathcal{J}$. is a function $E_{\mathcal{J}}: \mathcal{J} \to [0,1]$ is an entropy on INSs which satisfying:

i. $E_{\mathcal{J}}(N) = 0$ if N is crisp set
ii. $E_{\mathcal{J}}(N) = 1$ if $[t_N^l(y), t_N^u(y)] = [f_N^l(y), f_N^u(y)]$ and $[i_N^l(y), i_N^u(y)] = [i_{N^c}^l(y), i_{N^c}^u(y)]$ for all $y \in \mathfrak{U}$
iii. $E_{\mathcal{J}}(N) = E_{\mathcal{J}}(N^c)$ for all $N \in \mathcal{J}$.
iv. $E_{\mathcal{J}}(N) \geq E_{\mathcal{J}}(M)$ if $N \subseteq M$ when $i_N^l(y) + i_N^u(y) < 1$ and $i_M^l(y) + i_M^u(y) < 1$, for all $y \in \mathfrak{U}$.

Definition 3.5. Let N be an INS. Then the entropy of N is,

$$E_{\mathcal{J}}(N) = \frac{1}{n}\sum_{i=1}^{n} \left\{\frac{4 - |t_N^l(y_i) - f_N^l(y_i)| - |t_N^u(y_i) - f_N^u(y_i)|}{4 + |t_N^l(y_i) - f_N^l(y_i)| + |t_N^u(y_i) - f_N^u(y_i)|}\right.$$

$$\left.\frac{-\left|i_N^l(y_i)-i_{N^c}^l(y_i)\right|-\left|i_N^u(y_i)-i_{N^c}^u(y_i)\right|}{+\left|i_N^l(y_i)-i_{N^c}^l(y_i)\right|+\left|i_N^u(y_i)-i_{N^c}^u(y_i)\right|}\right\}$$

for all $y_i \in \mathfrak{U}$.

Theorem 3.6. The entropy of $E_J(N)$ is an entropy measure for IVN sets.

Proof: We show that the $E_J(N)$ satisfies the conditions $i-vi$ in Definition 3.4.

i. When N is a crisp set, i.e., $t_N^l(y_i)=t_N^u(y_i)=0$, $i_N^l(y_i)=i_N^u(y_i)=0$, $f_N^l(y_i)=f_N^u(y_i)=1$ or $t_N^l(y_i)=t_N^u(y_i)=1$, $i_N^l(y_i)=i_N^u(y_i)=0$, $f_N^l(y_i)=f_N^u(y_i)=0$, for all $y_i \in \mathfrak{U}$. It is clear that $E_J(N)=0$.

ii. Set $[t_N^l(y_i),t_N^u(y_i)]=[f_N^l(y_i),f_N^u(y_i)]=[a,b]$ and $[i_N^l(y_i),i_N^u(y_i)]=\left[i_{N^c}^l(y_i),i_{N^c}^u(y_i)\right]=[c,d]$ for all $y_i \in \mathfrak{U}$. Then

$$E_J(N)=\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{4-|a-a|-|b-b|-|c-c|-|d-d|}{4+|a-a|+|b-b|+|c-c|+|d-d|}\right\}=\frac{1}{n}\sum_{i=1}^{n}\frac{4}{4}=1.$$

iii. Since

$$t_{N^c}(y_i)=f_N(y_i)$$
$$f_{N^c}(y_i)=t_N(y_i)$$
$$i_{N^c}^l(y_i)=1-i_N^u(y_i)$$
$$i_{N^c}^u(y_i)=1-i_N^l(y_i)$$

for all $y_i \in \mathfrak{U}$, it is clear that $E_J(N)=E_J(N^c)$.

iv. If $N\subseteq M$, then

$$t_N^l(y_i)\le t_M^l(y_i);\ t_N^u(y_i)\le t_M^u(y_i)$$
$$i_N^l(y_i)\ge i_M^l(y_i);\ i_N^u(y_i)\ge i_M^u(y_i)$$
$$f_N^l(y_i)\ge f_M^l(y_i);\ f_N^u(y_i)\ge f_M^u(y_i)$$

for all $y_i \in \mathfrak{U}$. So

$$\left|t_N^l(y_i)-f_N^l(y_i)\right|\le\left|t_M^l(y_i)-f_M^l(y_i)\right|$$
$$\left|t_N^u(y_i)-f_N^u(y_i)\right|\le\left|t_M^u(y_i)-f_M^u(y_i)\right|$$

and, $i_N^l(y)+i_N^u(y)<1$ and $i_M^l(y)+i_M^u(y)<1$,

$$\left|i_N^l(y_i)-i_{N^c}^l(y_i)\right|\le\left|i_M^l(y_i)-i_{M^c}^l(y_i)\right|$$
$$\left|i_N^u(y_i)-i_{N^c}^u(y_i)\right|\le\left|i_M^u(y_i)-i_{M^c}^u(y_i)\right|$$

for all $y_i \in \mathfrak{U}$, then $E_J(N)\ge E_J(M)$.

The proof is completed.

Similarly, the weighted entropy measure for INSs is defined as follows:

$$E_{\omega J}(N)=\frac{1}{n}\sum_{i=1}^{n}\omega_i\left\{\frac{4-\left|t_N^l(y_i)-f_N^l(y_i)\right|-\left|t_N^u(y_i)-f_N^u(y_i)\right|}{4+\left|t_N^l(y_i)-f_N^l(y_i)\right|+\left|t_N^u(y_i)-f_N^u(y_i)\right|}\right.$$
$$\left.\frac{-\left|i_N^l(y_i)-i_{N^c}^l(y_i)\right|-\left|i_N^u(y_i)-i_{N^c}^u(y_i)\right|}{+\left|i_N^l(y_i)-i_{N^c}^l(y_i)\right|+\left|i_N^u(y_i)-i_{N^c}^u(y_i)\right|}\right\}$$

where $\omega=(\omega_1,\omega_2,\dots\omega_n)^T$ with $\omega_j\in[0,1]$, $\sum_{i=1}^{n}\omega_i=1$.

Here, we propose a method for multi-criteria decision method under SVN and IN environment.

Firstly, we apply our proposed entropy measure to MCDM with SVN information. The set of alternatives is denoted by $S=\{S_1,S_2,\dots,S_m\}$, and the set of attributes is denoted by $\mathcal{A}=\{\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n\}$. Let $\omega=(\omega_1,\omega_2,\dots\omega_n)^T$ be the probable weighting vector of the attribute $\mathcal{A}_j$ where $\omega_j\ge 0$, $\sum_{j=1}^{n}\omega_j=1$, $1\le j\le n$. Assume that $A=\left[a_{ij}\right]_{m\times n}$ is the decision matrix, where $a_{ij}=\left(t_{ij},\imath_{ij},f_{ij}\right)$ is characterized by SVN variable for an alternative S_i with respect to a criterion $\mathcal{A}_j$, and $0\le t_{ij}\le 1$, $0\le \imath_{ij}\le 1$, $0\le f_{ij}\le 1$, $t_{ij}+\imath_{ij}+f_{ij}\le 3$.

We now improve an approach for the decision maker to determine the perfect choice with SVN information. It is carried out the following steps to get best choice:

Step1. *The entropy values are computed corresponding to each alternative S_i $(i=1,2,\dots,m)$ by using the proposed entropy measure*

Step 2. *The alternatives are put in order according to the values of the entropy measures.*

Step 3. *The best alternative is selected in accordance with the value of entropy.*

Step4. *End.*

Example 3.7. Suppose that a food & beverage company that wants to select the best accounting software. There are four possible alternatives in which to choose the software program: S_1, S_2, S_3 and S_4. The food & beverage company must give a decision according to the three attributes: $\mathcal{A}_1$ is the price; $\mathcal{A}_2$ is the security, and $\mathcal{A}_3$ is the efficiency. Suppose that $\omega=(0.40,0.25,0.35)$ is weight vector of the attribute for TMD, the IMD and the FMD, respectively. The possible alternatives are computed with respect to these attributes. Decision makers provide the alternatives in the form of SVN according to the attributes $\mathcal{A}_j$ $(j=1,2,3)$. The SVN decision matrix A is obtained as follow:

$$A=\begin{pmatrix}\{0.6,0.2,0.1\} & \{0.3,0.1,0.3\} & \{0.1,0.3,0.4\}\\ \{0.2,0.3,0.2\} & \{0.4,0.1,0.2\} & \{0.4,0.3,0.2\}\\ \{0.6,0.0,0.2\} & \{0.4,0.3,0.1\} & \{0.4,0.2,0.3\}\\ \{0.4,0.2,0.3\} & \{0.5,0.1,0.2\} & \{0.3,0.3,0.4\}\end{pmatrix}.$$

If one needs to select the best alternative(s), one carry out the following steps:

Step 1. The weighted entropy measures of the alternatives are computed by the use of the entropy measure:

$$E_{\omega\mathcal{G}}(S_1) = 0.131, \qquad E_{\omega\mathcal{G}}(S_2) = 0.179,$$
$$E_{\omega\mathcal{G}}(S_3) = 0.120, \qquad E_{\omega\mathcal{G}}(S_4) = 0.158.$$

Step 2. According to the values of entropy measure, the alternatives are ordered as $S_2 \succ S_4 \succ S_1 \succ S_3$.

Step 3. The third alternative S_3 is the appropriate choosing with respect to the entropy values.

Secondly, we apply our proposed entropy measure to MCDM with IN information. The set of alternatives is denoted by $N = \{N_1, N_2, \dots, N_m\}$, and the set of attributes is denoted by $\mathcal{B} = \{\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_n\}$. Let $\omega = (\omega_1, \omega_2, \dots \omega_n)^T$ be the probable weighting vector of the attribute $\mathcal{B}_j$ where $\omega_j \geq 0, \sum_{j=1}^{n} \omega_j = 1, 1 \leq j \leq n$. Assume that $B = [b_{ij}]_{m\times n}$ is the decision matrix, where $b_{ij} = ([t_{ij}^l, t_{ij}^u], [i_{ij}^l, i_{ij}^u], [f_{ij}^l, f_{ij}^u])$ is characterized by IN variable for an alternative N_i with respect to a criterion $\mathcal{B}_j$, and $0 \leq t_{ij}^u + i_{ij}^u + f_{ij}^u \leq 3$, $t_{ij}^l \geq 0, i_{ij}^l \geq 0, f_{ij}^l \geq 0$.

We now improve an approach for the decision maker to determine the perfect choice with IN information. It is carried out the following steps to get best choice:

Step1. *The entropy values are computed corresponding to each alternative N_i $(i = 1,2,\dots,m)$ by using the proposed entropy measure*

Step 2. *The alternatives are put in order according to the values of the entropy measures.*

Step 3. *The best alternative is selected in accordance with the value of entropy.*

Step4. *End.*

Example 3.8. Suppose that a machine factory that wants to select the best selection of plot location. There are four possible alternatives in which to choose the location: N_1, N_2, N_3 and N_4. The machine factory must give a decision according to the three attributes: $\mathcal{B}_1$ is the proximity to markets; $\mathcal{B}_2$ is the proximity to suppliers, and $\mathcal{B}_3$ is the proximity to competitors. Suppose that $\omega = (0.25, 0.35, 0.40)$ is weight vector of the attribute for TMD, the IMD and the FMD, respectively. The possible alternatives are computed with respect to these attributes. Decision makers provide the alternatives in the form of IN according to the attributes $\mathcal{B}_j$ $(j = 1,2,3)$. The IN decision matrix D is obtained as follow:

$$\begin{bmatrix} \langle[0.4,0.8],[0.1,0.3],[0.1,0.2]\rangle & \langle[0.2,0.4],[0.2,0.5],[0.1,0.5]\rangle & \langle[0.0,0.2],[0.1,0.5],[0.3,0.5]\rangle \\ \langle[0.1,0.3],[0.2,0.4],[0.2,0.4]\rangle & \langle[0.2,0.5],[0.1,0.2],[0.3,0.8]\rangle & \langle[0.3,0.4],[0.0,0.5],[0.1,0.3]\rangle \\ \langle[0.3,0.5],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.1,0.4],[0.2,0.7],[0.1,0.2]\rangle & \langle[0.3,0.5],[0.3,0.4],[0.3,0.6]\rangle \\ \langle[0.1,0.5],[0.1,0.3],[0.1,0.5]\rangle & \langle[0.3,0.9],[0.0,0.2],[0.1,0.3]\rangle & \langle[0.1,0.5],[0.2,0.4],[0.1,0.7]\rangle \end{bmatrix}$$

If one needs to select the best alternative(s), one carry out the following steps:

Step 1. The weighted entropy measures of the alternatives are computed by the use of the entropy measure:

$$E_{\omega\mathcal{I}}(N_1) = 0.187, \qquad E_{\omega\mathcal{I}}(N_2) = 0.163,$$
$$E_{\omega\mathcal{I}}(N_3) = 0.176, \qquad E_{\omega\mathcal{I}}(N_4) = 0.148.$$

Step 2. According to the values of entropy measure, the alternatives are ordered as $N_1 \succ N_3 \succ N_2 \succ N_4$.

Step 3. The third alternative N_4 is the appropriate choosing with respect to the entropy values

4. Discussion and Conclusion

In this study, we define the entropy measures for SVNSs and INSs. A MCDM method is improved to illustrate the proposed entropy measure. Finally, the investment problem is solved.

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